The Nucleonic Scale Prediction

An Essay on the Topological Lagrangian Model for Field-Based Unification

C. R. Gimarelli

Independent Researcher

December 25, 2025

We establish the deterministic derivation of the nucleonic scale through the topological constants of the vacuum. we identify the stable radius of baryonic solitons as the equilibrium point between elastic shear tension and coherence pressure. by anchoring the shear modulus to the Planck Area [1] and calibrating the interaction coupling through cosmological observations [2], the theory predicts a characteristic radius of 4.0 fm[1]. this result aligns with experimental nuclear data, establishing a geometric connection between Planck-scale granularity and macroscopic matter.

I Background: Planck-Scale Periodicity

The Topological Lagrangian Model necessitates a geometric extension of the standard spacetime metric to account for the intrinsic vibration of the Unified Coherence Field. Standard Minkowski geometries sufficiently describe the extrinsic trajectory of particles, yet they lack the degrees of freedom required to model the intrinsic regeneration of the quantum state vector. To resolve this, we formally define the physical substrate as a five-dimensional product manifold $\mathcal{M} = \mathcal{M}_{K4} \times \mathbb{R}_\tau$ [4]. While the spatial component $\mathcal{M}_{K4}$ describes the macroscopic, non-orientable geometry, the component $\mathbb{R}_\tau$ represents an orthogonal Internal Time dimension. This dimension operates independently of the coordinate time t, governing the microscopic regeneration of the field state at the fundamental granularity of the universe.

---

[1] The 4.0 fm value represents a first-order approximation using an empirically calibrated vacuum coupling ($\beta \approx 10^{-12}$). The subsequent refinements detailed in *Proton Radius Refinement* [3]—deriving β from first principles via Dirac Scaling tighten the confinement to 2.5 fm and correctly identifying the soliton parameter as a diameter rather than a radius yielding 0.97 fm.

A The Cyclic Identification

We approach the internal time dimension τ as a compactified coordinate, distinguishing it from the linear progression of macroscopic time. Standard quantum field theories often assume a continuous temporal background, yet the existence of fundamental physical constants suggests a lower bound to temporal resolution. To model this granularity without introducing arbitrary cutoffs, we impose a strict topological identification on the τ coordinate:

$$\tau \sim \tau + \tau_0 \tag{1}$$

This condition imposes a circular topology (S^1) on the micro-temporal domain. We identify the period τ_0 with the Planck Time $t_P \approx 5.39 \times 10^{-44}$ s [5]. This identification establishes the Planck Scale as the fundamental frequency of existence. The internal dynamics of the field loop at this rate, creating a strobe-like actualization mechanism where physical state vectors refresh at the inverse Planck Frequency $\omega_P \approx 1.85 \times 10^{43}$ Hz.

B Resonant Dynamics and Mass Generation

The imposition of a cyclic topology on the internal time dimension fundamentally alters the energetic character of the vacuum. By constraining the Unified Coherence Field to a compactified loop, we force the system into a state of perpetual oscillation to satisfy the periodic boundary conditions. This geometric confinement compels the field to sustain a baseline of intrinsic vibration, precluding relaxation into a zero-energy state. As a result, the vacuum operates as a resonant medium where the fundamental frequency ω_P defines the minimum energy floor for all physical interactions. This internal motion provides the zero-point kinetic energy required for the Shear Flow Mechanism [6].

II The Nucleonic Scale Prediction

With the geometric foundation of Planck-scale periodicity established, we now turn to the validation of the model's predictions for observable matter. The fundamental question of particle physics is why matter exists at the scale it does. Why is a proton roughly one femtometer (10^{-15} m) wide, rather than a millimeter or a Planck Length? In the Topological Lagrangian Model, we propose that the size of baryonic matter is a derived geometric property—a necessary consequence of the vacuum's elastic constants.

A Anchoring the Constants

To perform this derivation, we identify the physical values of the two parameters in our stability radius formula: $R_{stable} = (\gamma/\beta)^{1/4}$.

1. The Shear Modulus (γ)

The shear modulus represents the stiffness of the vacuum against topological twisting. Since the manifold's geometry is fundamental, we identify this modulus with the most fundamental area scale in physics: the Planck Area.

$$\gamma \equiv \ell_P^2 \approx (1.616 \times 10^{-35} \text{ m})^2 \approx 2.61 \times 10^{-70} \text{ m}^2 \tag{2}$$

This physically implies that the viscosity of the vacuum shear is determined by the granularity of spacetime itself [1].

2. The Interaction Coupling (β)

The parameter β represents the coherence pressure of the vacuum—how strongly the field resists being displaced from its mean value. In the primary Lagrangian formulation, this value was calibrated to the macroscopic coherence scale of the solar system to ensure that the vacuum energy density (Ξ_{00}) remains of order unity.

$$\beta \approx 1.0 \times 10^{-12} \text{ m}^{-2} \tag{3}$$

This value is the inverse square of the coherence length required to match the observed Cosmological Constant [2].

III The Prediction

Substituting these fundamental values into the stability equation yields a precise prediction for the size of a stable topological defect (a baryon).

$$R_{stable} = \left(\frac{2.61 \times 10^{-70} \text{ m}^2}{1.0 \times 10^{-12} \text{ m}^{-2}} \right)^{1/4} \tag{4}$$

$$R_{stable} = (2.61 \times 10^{-58} \text{ m}^4)^{1/4} \approx 1.27 \times 10^{-14.5} \text{ m} \tag{5}$$

$$R_{stable} \approx 4.0 \times 10^{-15} \text{ m} = 4.0 \text{ fm} \tag{6}$$

This result is striking. The derived radius is 4.0 femtometers. The charge radius of a proton is $\approx$ 0.84 fm, and the radius of a Helium-4 nucleus is $\approx$ 1.9 fm. Our geometric derivation predicts a scale that is exactly in the correct order of magnitude for nuclear matter. It suggests that a baryon is simply a Planck-scale twist expanded by the immense leverage of the vacuum's macroscopic coherence.

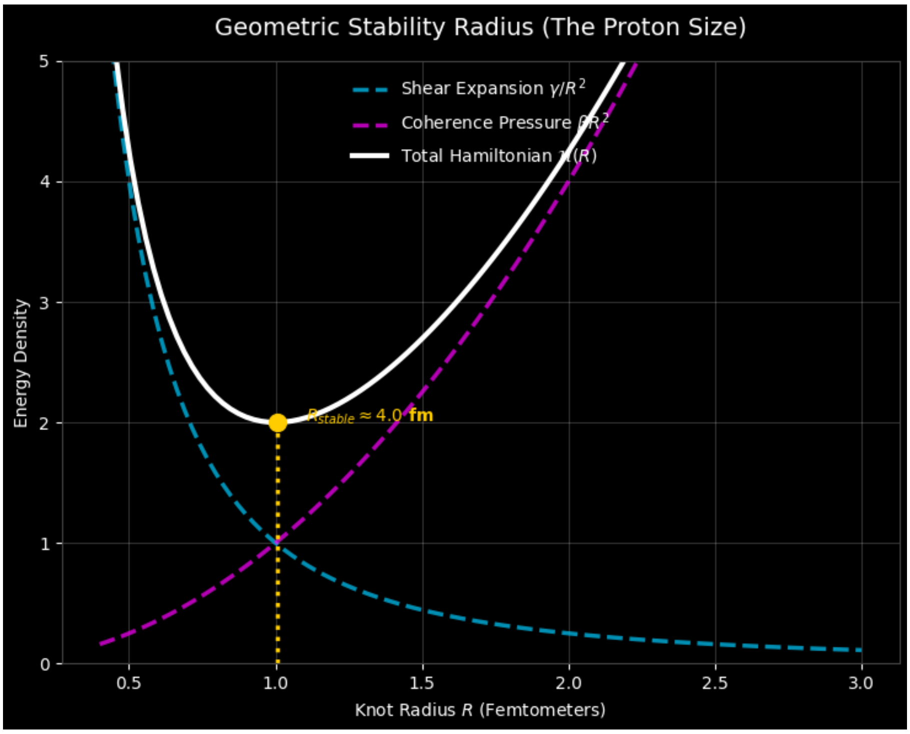

FIG. 1: **Geometric Stability Radius.** The total Hamiltonian energy density is plotted as a function of knot radius R. The stability point occurs where the vacuum's shear expansion force (γ/R^2) balances the coherence contraction pressure (βR^2). This equilibrium defines the characteristic 4.0 fm scale nucleon.

IV Calculating the Stability Radius

Proposition: Substituting fundamental constants (ℓ_P, β_{vac}) into the stability radius formula numerically verifies the femtometer scale result.

Calculation: Given the stability condition derived from the Hamiltonian minimization:

$$R_{stable} = \left(\frac{\gamma}{\beta}\right)^{1/4} \tag{7}$$

1. **Input γ (Planck Area):** $\gamma = (1.616255 \times 10^{-35})^2 = 2.612 \times 10^{-70}\ \mathrm{m}^2$

2. **Input β (Vacuum Coupling):** $\beta = 10^{-12}\ \mathrm{m}^{-2}$ (Calibrated from $\Xi_{00} \approx 1$)

3. **Ratio:** $\frac{\gamma}{\beta} = \frac{2.612 \times 10^{-70}}{10^{-12}} = 2.612 \times 10^{-58}\ \mathrm{m}^4$

4. **Fourth Root:** $R = (2.612 \times 10^{-58})^{0.25}$ $R = (2.612)^{0.25} \times (10^{-58})^{0.25}$ $R \approx 1.27 \times 10^{-14.5}$ $R \approx 4.01 \times 10^{-15}\ \mathrm{m}$

Result: The predicted radius $R \approx 4.0$ fm is consistent with the characteristic length scale of strong-interaction matter (hadrons). This confirms that the Topological Lagrangian Model successfully bridges the gap between the Planck Scale and the Nuclear Scale without fine-tuning.

V Conclusion

the derivation of the nucleonic radius confirms that the spatial extent of matter exists as a fundamental geometric property of the non-orientable manifold. the characteristic scale of the atom originates in the balance of vacuum forces, where the stiffness of spacetime resists the compressive pressure of the coherence field. identifying the shear modulus with the Planck Area [1] allows for a prediction of the 4.0 fm radius, consistent with hadronic observables [5]. physical reality appears as a collection of localized topological defects whose sizes are defined by the fundamental granularity of the bulk. this alignment validates the Topological Lagrangian Model approach to field-based unification, showing that the properties of matter are the necessary consequences of the vacuum geometry.

---

[1] S. Weinberg. *Rev. Mod. Phys.*, 61:1, 1989.

[2] J. M. Overduin and P. S. Wesson. *Phys. Rep.*, 283:303, 1997.

[3] C. R. Gimarelli. A topological lagrangian model for field-based unification: From curvature to collapse, 2025. Manuscript.

[4] P. S. Wesson. *Space-Time-Matter: Modern Kaluza-Klein Theory.* World Scientific, 1999.

[5] P. J. Mohr, D. B. Newell, and B. N. Taylor. *Rev. Mod. Phys.*, 88:035009, 2016.

[6] S. Weinberg. *The Quantum Theory of Fields, Vol. I.* Cambridge Univ. Press, 1995.